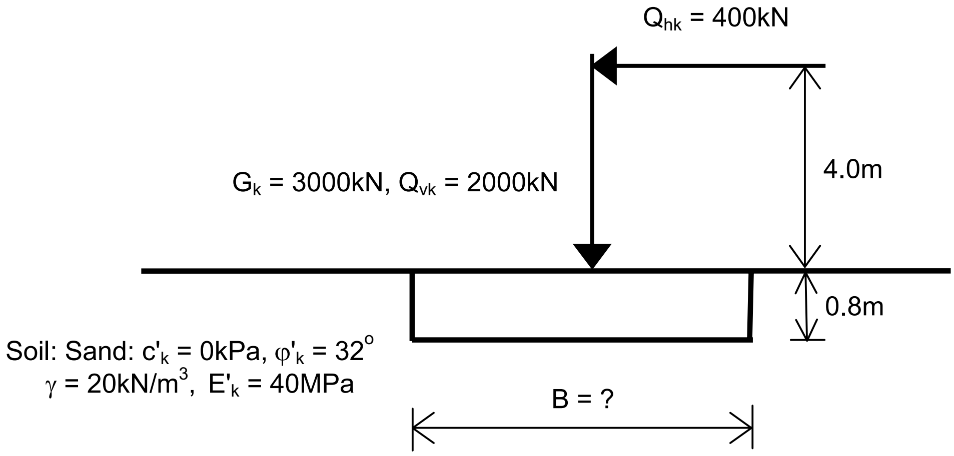
Model Solution for Example 2 – Pad Foundation with an Inclined Eccentric Load

1. Description of the problem



- **Design situation:**
  - Square foundation for a building, 0.8m embedment depth, groundwater level at great depth. Allowable settlement is 25mm and maximum rotation is 1/2000.
- **Soil conditions:**
  - Cohesionless sand, $c'_k = 0$, $\varphi'_k = 32°$, $\gamma = 20kN/m^3$, $E'_k = 40MPa$.
- **Characteristic values of actions:**
  - Permanent vertical load G_k = 3000kN plus weight of pad foundation;
  - Variable vertical load Q_{vk} = 2000kN (at top of foundation);
  - Permanent horizontal load = 0;
  - Variable horizontal load Q_{hk} = 400kN at a height of 4m above the ground surface;
  - Variable loads are independent of each other.
- **Require foundation width, B.**

2. Ultimate limit state design using an analytical method

2.1 – Bearing resistance

Bearing resistance failure is one ultimate limit state considered for the design of the pad. The following inequality needs to be satisfied *(6.5.2.1(1)P)*:

$$V_d \leq R_d$$

The minimum footing size is assessed for each *Design Approach* and just for drained conditions as $c'_k = 0$.

In this example, as the horizontal variable load is applied 4m above the ground surface, a moment *M* acts on the foundation, introducing an eccentricity *e*, and the effective area is different from the nominal area (i.e. $A' \neq A$). Furthermore, as the variable vertical and horizontal loads are independent of each other, according to EN 1990 *(6.4.3.2 Eqn. 6.10)* it is necessary to combine the leading and accompanying variable loads, with a combination factor of Ψ_o = 0.7 applied to the accompanying variable action, in order to determine the least favourable combination. The partial and combination factors are applied to loads at the start of the calculation in order to determine the load eccentricity and this eccentricity is then used to calculate the design resistance.

For each Design Approach, it is necessary to check if treating the vertical design load, V_d as a favourable action, which increases the eccentricity but reduces the vertical load, or as an unfavourable action, which reduces the eccentricity but increases the vertical load, is the most severe condition. Treating the Q_h as the leading variable load is the most severe condition.

$$V_d = G_d + G_{pad,d} + Q_{vd} = \gamma_G(Gk + B \times B \times h \times \gamma_{concrete}) + \gamma_Q \times \psi_o \times Q_{vk}$$

where G and Q_v are the permanent and variable vertical loads, G_{pad} is the weight of the pad foundation, B and h are the width and height of the foundation, $\gamma_{concrete}$ is the weight density of the foundation and the subscripts k and d indicate characteristic and design values. If the horizontal variable load is Q_h and this acts at a height h above the top of the foundation and the foundation is t thick, then the moment on the base of foundation is $M = Q(h + h)$. The eccentricity $e = M/V$ and the effective width $B' = B\text{-}2e$. To ensure that special allowances are not necessary, the eccentricity is checked to ensure that it does not exceed *B/3 (6.5.4(1)P)*.

The value of the design drained bearing resistance, R_d is calculated using *Eqn. D.2* of *Annex D:*

$$R_d/A' = c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + 0.5\ \gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma$$

where the bearing factors are:

$N_q = e^{\pi \tan \phi'} \tan^2(45 + \phi'/2)$

$N_c = (N_q - 1) \cot\phi'$

$N_\gamma = 2\ (N_q - 1) \tan\phi'$ — *when $\delta > \phi'/2$ (rough base)*

$s_q = 1 + \sin\phi'$ — *for a square or circular shape*

$s_\gamma = 1 - 0.3(B'/L')$ — *for a rectangular shape*

$s_\gamma = 0.7$ — *for a square or circular shape*

$s_c = (s_q N_q - 1)/(N_q - 1)$ — *for rectangular, square or circular shape*

$i_c = i_q – (1 - i_q)/N_c \tan\phi'$

$i_q = [1 - H/(V + A'c'\cot\phi')]^m$

$i_\gamma = [1 - H/(V + A'c'\cot\phi')]^{m+1}$

$m = [2 + (B'/L')]/[1+(B'/L')]$ — *when H acts in the direction of B'*

The *b* factors are all zero as the foundation base is horizontal. For DA1 and DA3, the R_d values are calculated by only applying partial factors to $\tan\phi'$ and the loads, while for DA2, R_d is calculated by applying partial factors of unity to $\tan\phi'$ and dividing the resistance obtained by γ_R. The values used in the design calculations for different design approaches and loading conditions are presented in Table 1.

The B values presented in Table 1 are the critical foundation widths when, for each design approach and loading condition, the design vertical load is equal to the design bearing resistance. The results show that, for the design conditions in this particular example, treating the vertical load as unfavourable is the critical loading condition for DA1 and DA3, while treating the vertical load as favourable is the critical loading condition for DA2. In the case of DA1, the design width is determined by Combination 2. Comparing the different design approaches, DA3 gives the largest width of 4.23m, DA2 gives the smallest width of 3.77m and DA1 gives an intermediate width of 3.98m. On overall factor of safety, OFS, is obtained by dividing the unfactored bearing resistance, R_k by the unfactored vertical load (with ψ= 1.0), V_k. For B = 3.98m, OFS = R_k / V_k = 2.5.

2.2 – Sliding resistance

The second ultimate limit state considered is sliding failure, satisfying the inequality *(6.5.3(2)P)*:

$$H_d \leq R_d + R_{p,d}$$

where H_d is the design horizontal load on the foundation, and R_d is the design horizontal resistance and $R_{p,d}$ is the passive earth resistance in front of the wall. As $R_{p,d}$ cannot be relied upon, it is assumed that $R_{p,d}$ = 0. The design shear resistance between the base of the wall and the soil, in front of the wall, R_d is given by:

$$H_d = (\ V_d' \tan\delta_d) / \gamma_R$$

Table 1: Ultimate limit state design of pad foundation

Parameter	DA1(C1)		DA1(C2)		DA2		DA3	
	$V_{fav.}$	$V_{unfav.}$	$V_{fav.}$	$V_{unfav.}$	$V_{fav.}$	$V_{unfav.}$	$V_{fav.}$	$V_{unfav.}$
B - pad width	3.46	3.26	3.97	**3.98**	**3.77**	3.65	4.09	**4.23**
γ_G-Structural (unfav.)	1.35	1.35	1.00	1.00	1.35	1.35	1.35	1.35
γ_G-Geotech (unfav.)	1.35	1.35	1.00	1.00	1.35	1.35	1.00	1.00
γ_G-Structural (fav.)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
γ_G-Geotech.(fav.)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
γ_Q-Structural (unfav.)	1.50	1.50	1.30	1.30	1.50	1.50	1.50	1.50
γ_Q-Geotech (unfav.)	1.50	1.50	1.30	1.30	1.50	1.50	1.35	1.35
Ψ_0-Comb. factor	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
G_{vk}	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00	3000.00
Q_{vk}	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00	2000.00
Q_{hk}	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
h - height of Q_h	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
d - pad depth	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80
γ_c – conc. wt density	24.00	24.00	24.00	24.00	24.00	24.00	24.00	24.00
G_{padk} = pad weight	230.49	203.97	302.99	303.94	272.18	255.26	320.63	344.02
V_d	3230.49	6425.36	3302.99	5123.94	3272.18	6494.6	3320.63	6614.43
Q_{hd}	600	600	520	520	600	600	600	600
Bearing Resistance								
M_d	2880	2880	2496	2496	2880	2880	2880	2880
e	0.892	0.448	0.756	0.487	0.880	0.443	0.867	0.435
Check B/3 – e > 0	0.26	0.64	0.57	0.84	0.37	0.77	0.49	0.98
B' = B-2e	1.68	2.36	2.46	3.00	2.00	2.76	2.35	3.36
L' = B	3.46	3.26	3.97	3.98	3.77	3.65	4.09	4.23
A' = B' x L	5.83	7.70	9.78	11.95	7.55	10.06	9.61	14.23
γ	20	20	20	20	20	20	20	20
q at foundation level	16	16	16	16	16	16	16	16
γ_ϕ	1.00	1.00	1.25	1.25	1.00	1.00	1.25	1.25
γ_c	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
γ_R	1.00	1.00	1.00	1.00	1.40	1.40	1.00	1.00
ϕ'_k	32	32	32	32	32	32	32	32
ϕ'd	32.00	32.00	26.56	26.56	32.00	32.00	26.56	26.56
N_q	23.177	23.177	12.588	12.588	23.177	23.177	12.588	12.588
N_γ	27.715	27.715	11.585	11.585	27.715	27.715	11.585	11.585
s_q	1.257	1.384	1.277	1.338	1.282	1.401	1.257	1.355
s_γ	0.854	0.783	0.814	0.773	0.840	0.773	0.827	0.762
m	1.673	1.580	1.617	1.570	1.653	1.569	1.635	1.557
i_q	0.709	0.857	0.758	0.845	0.716	0.859	0.851	0.862
i_γ	0.577	0.777	0.639	0.760	0.584	0.780	0.591	0.784
R_d	3230.49	6425.36	3302.99	5123.94	3272.18	6494.60	3320.63	6614.43
Vd	3230.49	6425.36	3302.99	5123.94	3272.18	6494.60	3320.63	6614.43
Check R_d-V_d > 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sliding Resistance								
$V_d = \gamma G_G k$	3230.491	3203.972	3302.99	3303.94	3272.18	3255.26	3320.63	3344.03
$H_d = \gamma_Q Q_h$	600	600	520	520	600	600	600	600
$\delta_d = \phi'_d$	32.0	32.0	26.6	26.6	32.0	32.0	26.6	26.6
$R_{hd} = V\tan\delta/\gamma_R$	2018.64	2002.06	1651.15	1651.62	1460.49	1452.94	1659.97	1671.66
R_d-V_d > 0	1418.64	1402.06	1131.15	1131.62	860.49	852.94	1059.97	1071.66

where δ_d is the design friction angle between the base of the foundation. According to *6.5.3(10)*, δ_d may be assumed equal to the design value of the effective critical state angle of shearing resistance $\phi'_{cv;d}$, for cast-in-situ concrete foundations and equal to 2/3 $\phi'_{cv;d}$ for smooth precast foundations". In this example it is assumed the foundation was cast-in-situ, so that δ_d=2/3 $\phi'_{cv;d}$. and it was assumed that $\phi'_{cv,d} = \phi'_d$.

For DA1 and DA3, γ_R = 1.0 so that the partial factors greater than zero are applied to *tan*ϕ' and the loads, while for DA2, the partial factor on *tan*ϕ' is unity so that partial factors greater than unity are applied to the loads and the calculated resistance is divided by γ_R.

The parameters values to check the sliding resistance are presented in Table 1 and the results show the foundation widths to ensure stability against bearing failure also to ensure the foundation is stable against sliding failure for all the design approaches and loading conditions.

3. Serviceability limit state design

To check that serviceability limit states are not exceeded, the following inequality must be satisfied:

$$E_d \leq C_d$$

where E_d is the calculated design value of the effect of the actions, i.e. the settlement or tilt, and C_d is the limiting design value of the effect of the actions, i.e. the maximum allowable settlement which is 25mm or the maximum allowable tilt, which is 1/2000 = 0.0005. In this example, as the direct method is used, a settlement calculation and a tilt calculation are used to check the serviceability limit states *(6.4(5)P)*.

3.1 – Settlement

As the soil is sand and only the Young's modulus, E is given, the settlement is calculated using the following elastic equation (Poulos and Davis, 1974):

$$s = \frac{\mathrm{V_k}(1-\nu^2)}{\mathrm{E}\beta_z\sqrt{\mathrm{BL}}}$$

where V_k is the net characteristic bearing pressure on the foundation due to the vertical loads, ν is Poisson's ratio and is assumed to be 0.3 and β_z is a coefficient whose value is 1.1 for a square foundation. Substituting the relevant values in the above equation for the smallest ULS design width of 3.77m for DA2 gives:

$$s = \frac{5272.9(1-0.3^2)}{40000 \text{ x } 1.1\sqrt{3.77 \text{ x } 3.77}} = 28.4\text{mm} > 25\text{mm}$$

As s = 28.4mm > 25mm, the settlement SLS condition is not satisfied for any of the ULS design widths.

3.2 – Tilt

According to *6.6.2(15)*, (15) the tilting of an eccentrically loaded foundation should be estimated by assuming a linear bearing pressure distribution and then calculating the settlement at the corner points of the foundation, using the vertical stress distribution in the ground beneath each corner point As an alternative to this application rule, the tilt is calculated using the elastic equation (Poulos and Davis, 1974) so that this design solution is not claimed to be wholly in accordance with EC7 (*1.4(4)*):

$$\theta = \frac{\mathrm{M_k}(1-\nu^2)}{\mathrm{EB^2L}} I_t = 27.6\text{mm} > 25\text{mm}$$

where M_k is the unfactored moment on the foundation base and I_t is a factor whose value is 3.7 for a square foundation when B = L. For B = 7.0m, the tilt is:

$$\theta = \frac{400 \text{ x } 4.8 \text{ x } (1-0.3^2)}{40000 \text{ x } 7.0^2 \text{ x } 7.0} 3.7 = 0.000471 = 1/2122 < 1/2000$$

Since θ = 1/2122 < 1/2000, the tilt SLS condition is satisfied when B = 7.0m. As B = 7.0m is larger than the ULS and settlement design widths, the foundation design in this example is controlled by the severe SLS requirement that the tilt should not exceed 1/2000.

References

Poulos H.G. & Davies E.H. (1974) Elastic solutions for soil and rock mechanics, Wiley